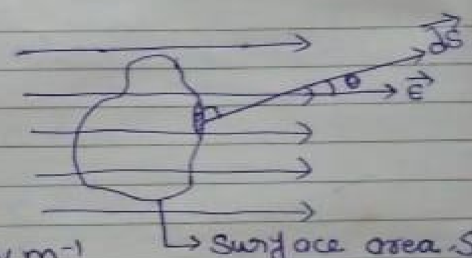
Electric flux

The electric flux may be defined as the total number of electric lines of force crossing through a surface ($\vec{S}$) placed in a uniform electric field ($\vec{E}$), normally to the surface. It is a scaler quantity & Mathematically it is equal to the surface integral of the electric field ~~through~~ over a surface. It is given as.

Electric flux,

$$\phi_E = \phi = \oint \vec{E}\cdot\vec{dS}$$

SI unit $= N C^{-1} m^2$
or $V m^{-1}$



Note:- Total Electric flux through a surface

$$\boxed{\phi = \vec{E}\cdot\vec{S} = ES\cos\theta}$$

$$\boxed{d\phi = \vec{E}\cdot\vec{dS} = E\,dS\cos\theta}$$

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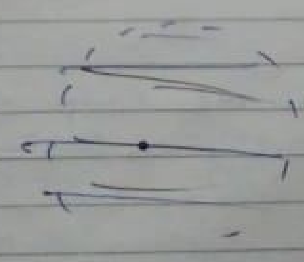
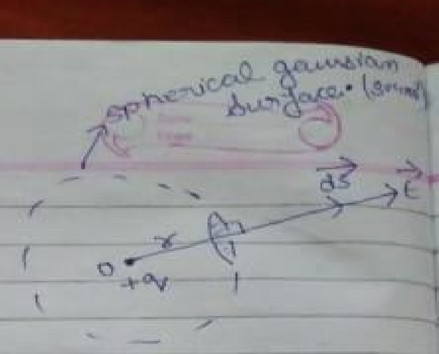
Gauss law

- It is the limitations of the coulomb's law in electrostatic.
- To determine the electric field or electric force due to a source ~~for~~ charge, by using Gauss law, a surface must be imagine surrounding a source charge, which is symmetrical and enclosed the source charge completely and also must passed through the reqd. point. This is called the gaussian surface.
- Gauss law states that "the total electric flux crossing through a closed surface, enclosing the charge completely is $\left(\frac{1}{\epsilon_0}\right)$ times charge enclosed by the Gaussian surface."

Types of charge	Gaussian surface
1) point charge, spherical shell, solid sphere.	Spherical
2) line charge, plane sheet of charge	cylindrical

Proof





$$d\phi = E\,ds\cos 0^\circ$$
$$= E\,ds$$
$$= \frac{kq}{r^2}(ds)$$

$$\phi_E = \int_{S=4\pi r^2} d\phi$$

$$= \frac{kq}{r^2}\int_0^{4\pi r^2} ds$$

$$= \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q}{r^2} \times 4\pi r^2$$

$$= \frac{q}{\epsilon_0}$$

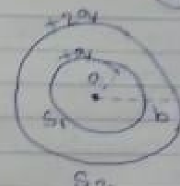
$$\boxed{\phi_E = \int_S \vec{E}\cdot ds = \frac{q}{\epsilon_0}}$$

where $q = q_{in}$ = Total charge enclosed by the spherical gaussian surface.

Ques:- A square loop of side 10cm is placed vertically ~~facing~~ east-west direction, in a uniform electric field of $10\,NC^{-1}$ directed in SW direction. calculate the total electric flux for the square loop.

~~We have,~~
~~$S = a^2 = (0.1)^2 = 0.01$~~
~~$E = 10 NC^{-1}$~~

2. Two concentric spherical plates S_1 & S_2, carring charge $+q$ & $+2q$ respectively as shown in figure.
Determine the ratio of ~~f~~ Electric flux obtained through two surfaces.



$b > a$

3. A surface $\vec{S} = (\hat{i} - 2\hat{j} + 3\hat{k})\, m^2$ is placed in a uniform E-field, $\vec{E} = (2\hat{i} - \hat{j} + \hat{k})\, NC^{-1}$. calculate the ~~net~~ E Total Electric flux to the given surface.

2. We have,

$$q_1 = +q$$
$$q_2 = +2q$$

$$\phi_1 = \frac{q_1}{\epsilon_0} = \frac{q}{\epsilon_0}$$

$$\phi_2 = \frac{q_2}{\epsilon_0} = \frac{3q}{\epsilon_0}$$

$$\therefore \frac{\phi_1}{\phi_2} = \frac{q/\epsilon_0}{3q/\epsilon_0} = \frac{1}{3} = 1:3$$

$$\phi_1 : \phi_2 = 1:3$$

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3 $\vec{S} = (\hat{i} - 2\hat{j} + 3\hat{k})\ m^2$
$\vec{E} = (2\hat{i} - \hat{j} + \hat{k})\ NC^{-1}$

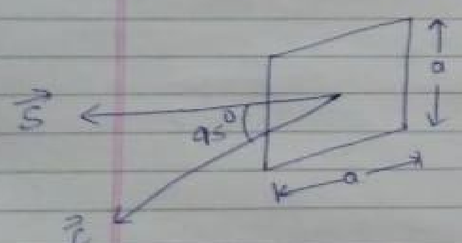
$$\vec{\phi} = \vec{E}\cdot\vec{S}$$

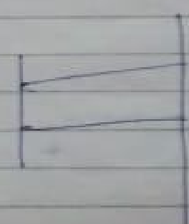
$$= (2\hat{i} - \hat{j} + \hat{k})\cdot(\hat{i} - 2\hat{j} + 3\hat{k})$$

$$|\vec{\phi}| = 2 + 2 + 3$$
$$= 7\ NC^{-1}\ m^2$$

1 we have,
$a = 1cm = 0.1m$
$S = a^2 = 1\times10^{-2}\ m$ (EW)

$E = 10\ N/C$ (SW)





$$\phi = ES\cos\theta$$
$$= 10\times1\times10^{-2}\cos45$$

$$= \frac{10^{-1}}{\sqrt{2}}$$

$$= \frac{1}{10\sqrt{2}}\ N/cm^2$$

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NCERT

4. we have,

$E = \alpha x^{1/2}$

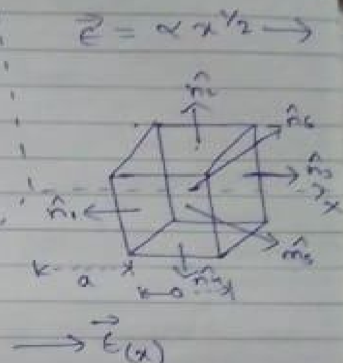
where,

α = arbitary const.

$\alpha = 800\ NC^{-1} m^{-1/2}$

$a = 10cm$

$= 0.1m$



(i) Total flux through the cube = ?

(ii) Total charge enclosed within the cube = ?

$\Phi = ES\cos\theta$ — ①

$\vec{S}_2, \vec{S}_4, \vec{S}_5$ & $\vec{S}_6$ are $\perp^r$ to $\vec{E}_x$ i.e. $\theta = 90^\circ$

$\cos 90^\circ = 0$

from (i)

$\Phi_2 = \Phi_4 = \Phi_5 = \Phi_6 = E_x \cos 90^\circ$

$= 0$

$\Phi_1 = E_1 S_1 \cos 180^\circ$ $[\vec{S}_1 \nparallel \vec{E}_1]$

$- E_1 S$

$- \alpha (0.1)^{1/2} \cdot 10^{-2}$ V/m

$\Phi_3 = E_3 S_3 \cos 0^\circ$ $[\vec{S}_1 \parallel \vec{E}_1]$

$+ E_3 S$

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$= \alpha (0.2)^{1/2} \times 10^{-2}$ V-m

$\phi_{total} = \phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5 + \phi_6$

$\phi_1 + \phi_3 \qquad [\phi_2 = \phi_4 = \phi_5 = \phi_6 = 0]$

$-\alpha (0.1)^{1/2} 10^{-2} + \alpha (0.2)^{1/2} 10^{-2}$

$\alpha 10^{-2} \left((0.2)^{1/2} - (0.1)^{1/2} \right)$

$800 \times 10^{-2} \left(\left(\frac{2}{10}\right)^{1/2} - \left(\frac{1}{10}\right)^{1/2} \right)$

$\frac{8}{\sqrt{10}} (\sqrt{2} - 1)$

$= \cancel{\frac{8}{\sqrt{10}}} \quad \frac{8\sqrt{10}}{10} \cdot 0.414$

$= \frac{4\sqrt{10}}{5} \; 0.414$

$\phi_{total} \pm \left(\frac{1.656\sqrt{10}}{5} \right)$ V-m

By Gauss law,

$\phi_{t} = \frac{q_{in}}{\epsilon_0}$

$q_{in} = \left(\frac{1.655\sqrt{10}}{5} \times 8.85 \times 10^{-12} \right) C$

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Q. why we calculate E-field of uniform body?
→ (i) density is uniform.
(ii) we can assume easily part.

* By using Gauss's law to find E-field at a point -

(1) A uniformly charged solid sphere

For a solid sphere (uniformly charged)

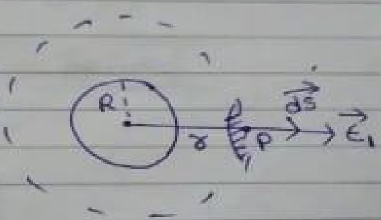
R = radius

ρ = volume charge density

$$= \frac{q}{\frac{4}{3}\pi R^3} = \frac{dq}{dV} \quad \text{---(1a)}$$

r = position of the reqd. point from the centre of the sphere.

case-I when $r > R$ (outside the sphere)



By Gauss's law

$$\phi_e = \oint_{S=4\pi r^2} E_1 dS \cos 0^\circ = \frac{q}{\epsilon_0}$$

$$= E_1 \times 4\pi r^2 = \frac{\rho \times \frac{4}{3}\pi R^3}{\epsilon_0}$$

$$\boxed{E_1 = E_{out} = \left(\frac{\rho R^3}{3\epsilon_0}\right)\frac{1}{r^2}} \quad \left[E_1 \propto \frac{1}{r^2}\right]$$

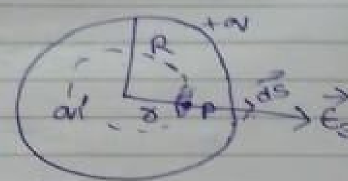
case-2 when $r = R$

from (2a)

$$E_2 = E_{surface} = \left(\frac{\rho R^3}{3\epsilon_0}\right)\frac{1}{R^2}$$

$\cancel{E_2 \propto r}$ $$\boxed{E_2 = E_{surface} = \frac{\rho R}{3\epsilon_0}}$$ Max^m & const.

case-3 when $r < R$



$$\rho = \frac{q'}{\frac{4}{3}\pi r^3}$$

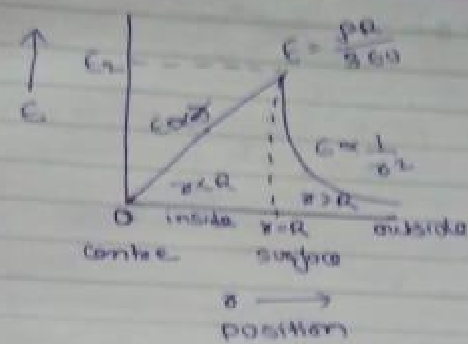
By Gauss's law,

~~from 2a~~ $$E_3 \times 4\pi r^2 = \frac{\rho \times \frac{4}{3}\pi r^3}{\epsilon_0}$$

$$\boxed{E_3 = \frac{\rho r}{3\epsilon_0}} \quad \text{— (2c)} \quad \boxed{E_3 \propto r}$$

If $r = 0$, i.e. $P \to 0$ (centre)
from 2c

$$\boxed{E_4 = 0}$$



(2) <u>A uniformly charged shell (Hollow sphere)</u>

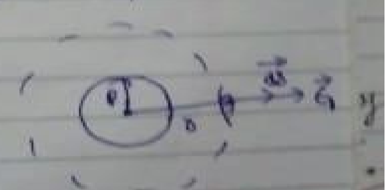
R = radius

σ = surface charge density

$= \frac{q}{4\pi R^2} = \frac{dq}{dA}$

r = position of the read. point for the centre of the sphere.

<u>case-I</u>, when $r > R$, outside the sphere.



<u>By Gauss's law</u>

$$\Phi_E = \oint_{4\pi r^2} E\,.ds\cos 0 = \frac{q}{\epsilon_0}$$

$$= \underline{E\,ds} = \frac{q}{\epsilon_0}$$

$$\cancel{E,\ 4\pi R^2} = \cancel{q}$$

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$$E_1 \times 4\pi r^2 = \frac{\sigma 4\pi R^2}{\epsilon_0}$$

$$\boxed{E_1 = \left(\frac{\sigma R^2}{\epsilon_0}\right)\frac{1}{r^2}} \quad \text{—(20)}$$

$$E_1 \propto \frac{1}{r^2}$$

(ii) $r = R$ i.e. E at surface.

$$\boxed{E_2 = E_{surface} = \frac{\sigma}{\epsilon_0}\frac{R^2}{R^2} = \frac{\sigma}{\epsilon_0}}$$ Max^m & const.

(iii) $r < R$ i.e. E at inside.

$q' = 0$ [as there is no mass inside the body]

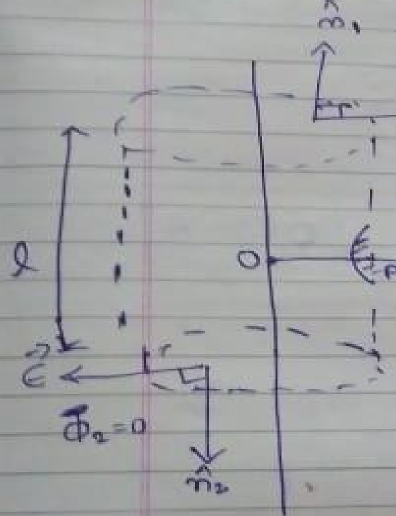
∴ By Gauss law

$$\phi_E = \oint E_3 ds \cos 0^\circ = \frac{q'}{\epsilon_0}$$

$$= E_3 4\pi r^2 = 0$$

$$\boxed{E_3 = 0}$$

(3) <u>A line charge</u> (a uniformly charged thin wire of infinite length).



λ = linear charge density

$$= \frac{q}{l} = \frac{dq}{dl} \quad \text{—(10)}$$

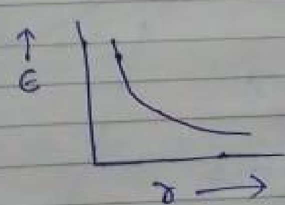
By Gauss's law

$$\phi_E = \oint_{S=2\pi r l} E \, dS \cos 0^\circ = \frac{q}{\epsilon_0}$$

$$= 2\pi r l \times E = \frac{\lambda l}{\epsilon_0}$$

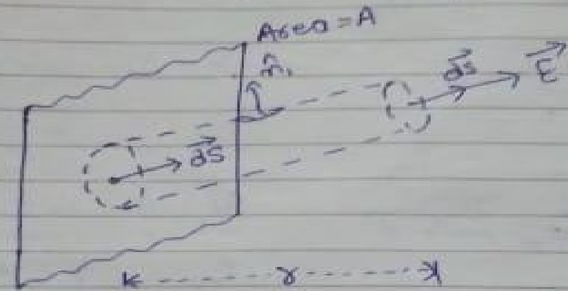
$$E = \frac{\lambda}{2\pi r \epsilon_0}$$

$$\boxed{E = \left(\frac{2\lambda}{4\pi\epsilon_0}\right)\frac{1}{r}}$$



(4) Sheet of charge (uniformly charged)

Let σ = Surface charge density $= \frac{dq}{dA}$



$$\boxed{\oint ds = A}$$

By Gauss' law

$$\phi_E = \oint_{S=2A} E\,ds\cos 0^\circ = \frac{q}{\epsilon_0}$$

$$\boxed{E \times 2A = \frac{q}{\epsilon_0}} \quad \text{— (1)}$$

for an infinite plane sheet of charge for a conducting sheet of charge

$$\sigma = \frac{q}{2A} \Rightarrow q = \sigma 2A$$

$$E \times 2A = \frac{\sigma 2A}{\epsilon_0}$$

$$\boxed{E = \frac{\sigma}{\epsilon_0}} = \boxed{\vec{E} = \left(\frac{\sigma}{\epsilon_0}\right)\hat{n}} \quad (2)$$

(Non-conducting)

for an infinite plane sheet of charge

$$\sigma = q/A \qquad q = \sigma A$$

$$E \times 2A = \frac{\sigma A}{\epsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\epsilon_0}}$$

$$= \boxed{\vec{E} = \left(\frac{\sigma}{2\epsilon_0}\right)\hat{n}}$$

* from eqn. (2) it is clear that for an infinite plane sheet of charge (conducting or non-conducting) always produces a uniform electric field which is independent on both dimension and position. ~~of the sl~~

Q. State and prove ~~coto~~ coulomb's law from Gauss law.

$$d\phi = \vec{E}\cdot\vec{dS}$$
$$= E\,dS\cos 0^\circ$$
$$= E\,dS$$

$$\phi = \oint_{S=4\pi r^2} \vec{E}\,\vec{dS}$$

$$= E\oint_{4\pi r^2} dS = E \times 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$E \times 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$E = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q}{r^2}$$

if a point charge q_0 is placed on the gaussian surface

$$F = q_0 E$$

$$\boxed{F = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q\cdot q_0}{r^2}}$$ — coulomb's law