MATRICES

MATRICES AND THEIR REPRESENTATIONS

Suppose we wish to express that Rohan has 5 pencils. We may express it as [5] or (5) with the understanding that the number inside [ ] denotes the number of pencils that Rohan has. Next suppose that we want to express that Rohan has 2 books and 5 pencils. We may express it as [2 5] with the understanding that the first entry inside [ ] denotes the number of books; while the second entry, the number of pencils, possessed by Rohan.

Let us now consider, the case of two friends Sohan and Mohan. Sohan has 3 books, 4 notebooks and 2 pens; and Mohan has 7 books, 5 notebooks and 4 pens.

A convenient way of representing this information is in the tabular form as follows:

	Books	Notebooks	Pens
Sohan	3	4	2
Mohan	7	5	4

We can also briefly write this as follows:

$$\begin{array}{cccc} & \text{First Column} & \text{Second Column} & \text{Third Column} \\ & \Downarrow & \Downarrow & \Downarrow \\ \text{First Row} & 3 & 4 & 2 \\ \text{Second Row} & 7 & 5 & 4 \end{array}$$

This representation gives the following information:

(1) The entries in the first and second rows represent the number of objects (Books, Notebooks, Pens) possessed by Sohan and Mohan, respectively

(2) The entries in the first, second and third columns represent the number of books, the number of notebooks and the number of pens, respectively.

Thus, the entry in the first row and third column represents the number of pens possessed by Sohan. Each entry in the above display can be interpreted similarly

The above information can also be represented as

	Sohan	Mohan
Books	3	7
Notebooks	4	5
Pens	2	4

which can be expressed in three rows and two columns as given below:

$$\begin{bmatrix}3 & 7\\4 & 5\\2 & 4\end{bmatrix} \text{ or } \begin{pmatrix}3 & 7\\4 & 5\\2 & 2\end{pmatrix}$$

Definition

A matrix is a rectangular array or arrangement of entries or elements displayed in rows and columns put within a square bracket or parenthesis. The entries or elements may be any kind of numbers (real or complex), polynomials or other expressions. Matrices are denoted by the capital letters like A, B, C…

Here are some examples of Matrices.

$$A = \begin{array}{cc} \begin{bmatrix}1 & 4\\2 & 5\\3 & 6\end{bmatrix} & \begin{array}{l}\textbf{First Row}\\\textbf{Second Row}\\\textbf{Third Row}\end{array} \\ \begin{array}{cc}\textbf{First Column} & \textbf{Second Column}\end{array} & \end{array} \qquad B = \begin{array}{cc} \begin{bmatrix}1 & -4 & 2\\6 & 9 & 4\\3 & -2 & 6\end{bmatrix} & \begin{array}{l}\textbf{First row } (R_1)\\\textbf{Second row } (R_2)\\\textbf{Third row } (R_3)\end{array} \\ \begin{array}{ccc}\textbf{First Column} & \textbf{Second Column} & \textbf{Third Column}\\ C_1 & C_2 & C_3\end{array} & \end{array}$$

Note : In a matrix, rows are counted from top to bottom and the columns are counted from left to right.

i.e. (i) The horizontal arrangements are known as rows.

(ii) The vertical arrangements are known as columns.

To identify an entry or an element of a matrix two suffixes are used. The first suffix denotes the row and the second suffix denotes the column in which the element occurs.

From the above example the elements of A are $a_{11} = 1$, $a_{12} = 4$, $a_{21} = 2$, $a_{22} = 5$, $a_{31} = 3$ and $a_{32} = 6$

Note: Plural of matrix is matrices

Order or size of a matrix

The order or size of a matrix is the number of rows and the number of columns that are present in a matrix.

In the above examples order of A is 3×2, (to be read as 3-by-2) and order of B is 3×3, (to be read as 3-by-3).

In general a matrix A of order $m \times n$ can be represented as follows :

$$A=\begin{bmatrix} a_{11} & a_{12} & ..a_{1j}.. & ..a_{1n} \\ a_{21} & a_{22} & ..a_{2j}.. & ..a_{2n} \\ a_{i1} & a_{i2} & ..a_{ij}.. & ..a_{in} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & ..a_{mj} & ..a_{mn} \end{bmatrix}$$

This can be symbolically written as $A = [a_{ij}]_{m\times n}$

The element aij belongs to ith row and the jth column. i being the row index and j being the column index. The above matrix A is an $m \times n$ or m-by-n matrix.

ILLUSTRATIVE EXAMPLES:

EXAMPLE: 1 A matrix has 8 elements. What are the possible orders it can have?

SOLUTION: All possible ordered pairs are

$(1, 8)$, $(8, 1)$, $(2, 4)$, $(4, 2)$

Hence, possible orders are 1×8, 8×1, 2×4, 4×2.

EXAMPLE: 2 If $A = [a_{ij}] = \begin{bmatrix} 2 & 3 & 4 \\ 4 & -2 & 3 \\ 0 & 7 & 2 \end{bmatrix}$ and

$B = [b_{ij}] = \begin{bmatrix} 2 & 4 & 6 \\ -3 & 1 & 5 \end{bmatrix}$, then find

(i) $a_{22} + b_{21}$ (ii) $a_{11} b_{21} + a_{32} b_{22}$

SOLUTION: (i) Here, $a_{22} = -2$, $b_{21} = -3$

Therefore $a_{22} + b_{21} = (-2) + (-3) = -5$

(ii) Here $a_{11} = 2$, $a_{32} = 7$, $b_{21} = -3$, $b_{22} = 1$.

Therefore. $a_{11} b_{21} + a_{32} b_{22} = (2)(-3) + (7)(1)$

$= -6 + 7$

$= 1.$

EXAMPLE: 3 Construct a 2×3 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = \dfrac{i+3j}{2}$

SOLUTION: Here $a_{ij} = \dfrac{i+3j}{2}$ (Given)

In general a 2×3 matrix is given by

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

Now, $a_{11} = \dfrac{1 + 3 \times 1}{2} = 2$, $a_{12} = \dfrac{1 + 3 \times 2}{2} = \dfrac{7}{2}$

$a_{13} = \frac{1 + 3\times3}{2} = 5$, $a_{21} = \frac{2+3\times1}{2} = \frac{5}{2}$

$a_{22} = \frac{2 + 3\times2}{2} = 4$, $a_{23} = \frac{2+3\times3}{2} = \frac{11}{2}$

Hence the required matrix is given by

$$A = \begin{bmatrix} 2 & \frac{7}{2} & 5 \\ \frac{5}{2} & 4 & \frac{11}{2} \end{bmatrix}$$

- : CHECK YOUR PROGRESS : -

(1.) Marks scored by two students A and B in three tests are given in the adjacent table. Represent this information in the matrix, in two ways.

	Test 1	Test 2	Test 3
A	49	60	75
B	61	48	70

(2.) In family M, there are 5 men, ~~and~~ 6 Women and 3 children and in family N, there are 4 men, 6 women and 5 children. Express this information in the form of matrix of order 3×2.

(3) What are possible orders of a matrix if it has (i) 14 elements, (ii) 18 elements

(4) How many elements in all are in a matrix of order (a) 2×4 (b) 5×3 (c) 1×6, (d) 4×6.

(5.) In a matrix $A = \begin{bmatrix} 4 & -2 & \sqrt{3} & \frac{1}{3} \\ 0 & 1 & 6 & -5 \\ 2\sqrt{3} & 6 & 0 & 7 \end{bmatrix}$, Find (a) Total number of elements.

(b) a_{21}, a_{32}, a_{31}, a_{14}, a_{23}.

(c) $a_{31} + a_{13} - a_{24}$

(6) Construct a matrix of order 4×3, whose elements are given by

(i) $a_{ij} = \frac{(2i-j)^2}{2}$ and (ii) $b_{ij} = \frac{i+j}{2i+3j}$

(7) Construct a 3×3 matrix $A = [a_{ij}]$ whose elements are given by

(i) $a_{ij} = \frac{1}{3}\left| -3i + 2j \right|$, (ii) $a_{ij} = \frac{(2i+j)^2}{|i+j|}$