* Electric lines of force or electric field lines :-

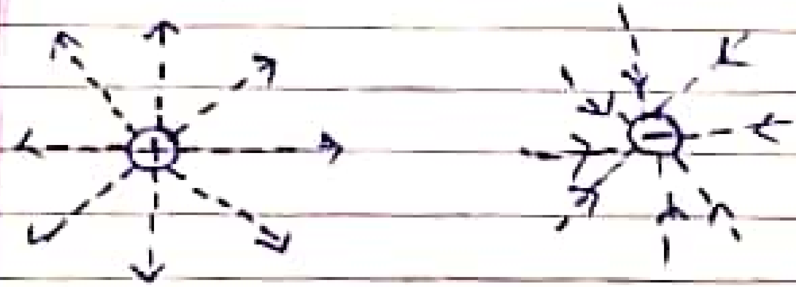
→ When a unit positive test charge is allowed to move freely inside an electric field, due to coloumb force acting on the test charge, it follows a continous path which is known as electric line of force.

The physical significance of the electric lines of force lies in the fact that the concept of electric field is real while that of electric lines of force are imaginary.

In other words, the region having ~~th~~ a net non-zero electric field must possessed an electric field lines while the converse, may or maynot be true.

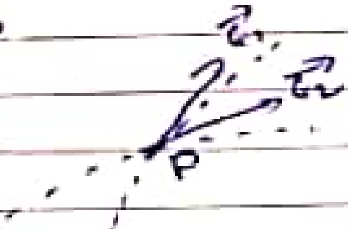
Properties :-

1. It may be ~~stretch~~ straight or curve
2. It starts from +ve charge and to the negative charge radially (normally to the surface)

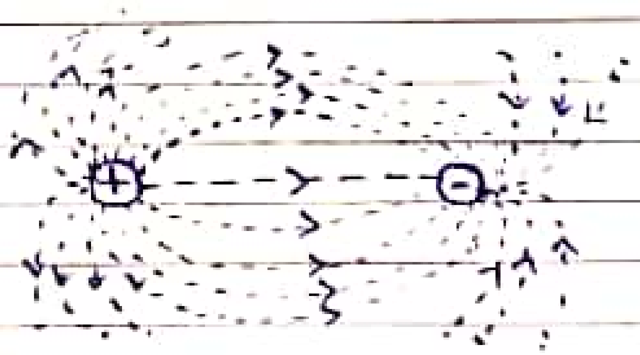


3. At a point on the Electric files, Accordy to Electric field is always directed along the ~~base~~ tangent drawn at that point in the direction of the electric field lines

4. Two electric field lines ~~reason~~ never intersect each other. If they ~~intersect each other~~ intersect then at the point of intersection, the electric field which has 2 different direction. which is practically impossible for a vector.



5. The Electric field ~~line~~ contract ~~ea~~ ~~pa~~ longitudinally (length wise) provided the unlike charge are brought ~~near~~ ~~to~~ near to each other. It is due to this reason, ~~unlike~~ the two unlike charges always attract each other.



16

6. Two electric-field lines exert lateral pressure on each other provided two like charges are brought near to each other. It is due to this reason two like charges generally repel each other.

B/w two like charges ($+q_1$ & q_2)

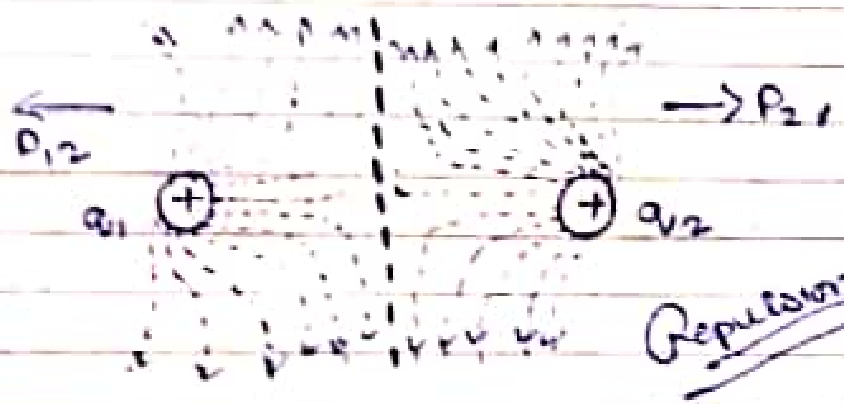
case 1 If $|q_1| = |q_2|$

Let P_{12} = pressure on q_1 due to q_2.

P_{21} = pressure on q_2 due to q_1.

then

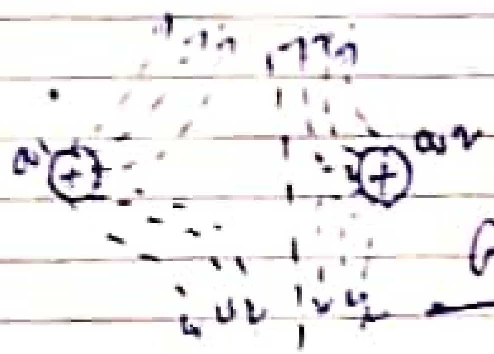
$P_{21} = P_{12}$. the lines of symmetry passes through the mid-point of the line joining q_1 & q_2.



Repulsion

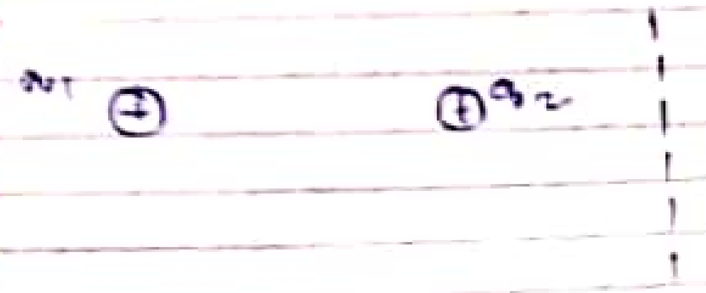
case 2 $|q_1| > |q_2|$

$|P_{21}| > |P_{12}|$



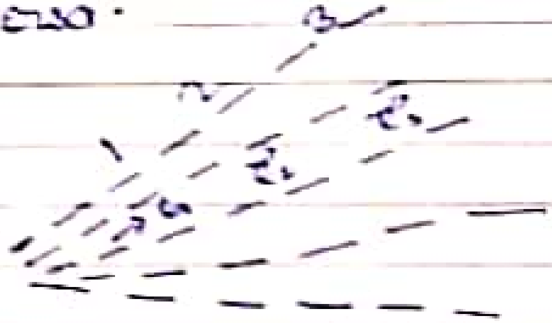
Repulsion

case-III $\quad$ if $|q_1| >>>> |q_2|$



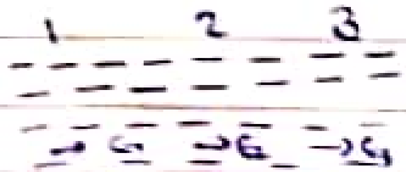
Between two like charges, If the difference in their magnitude is much higher. In this case the line of symmetry crosses the smaller charge & thus the greater charge traps the smaller charge in its Electric-field. This is the case of attraction between two like charges.

7. The Relative closeness of electric field lines determine the strength of electric field in that region. The region where electric field lines are more crowded (less spacing between field lines) stronger is the electric field in that region and Vice-versa.



$\vec{E_1} > \vec{E_2} > \vec{E_3}$

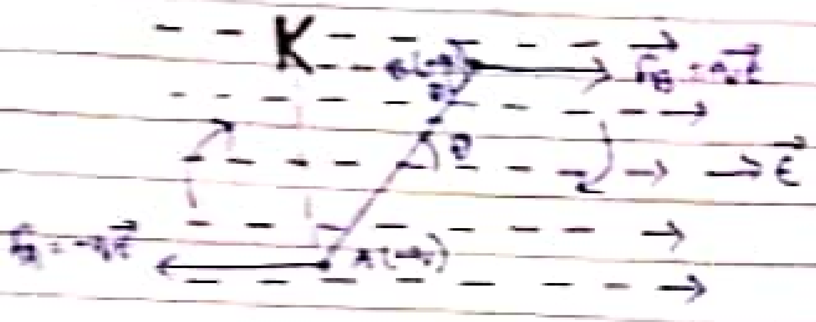
Non-uniform E-field



$\vec{E_1} = \vec{E_2} = \vec{E_3}$

uniform - E - field

* An electric dipole placed inside a uniform electric field obliquely



For an electric dipole, we have,
electric dipole moment is -

$$\vec{p} = q(2l)$$

θ = angle b/w $\vec{p}$ & $\vec{E}$

AB = 2l

∴ $|\vec{F_A}| = |\vec{F_B}|$

But

$\vec{F_A}$ ∥ $\vec{F_B}$, at θ = 180°

∴ $\vec{F_A} = -\vec{F_B}$

$\boxed{\vec{F_A} + \vec{F_B} = 0}$ — (13) Thus the electric is in translational equilibrium (only)

∵ $\vec{F_A} = -\vec{F_B}$ but having different line of action.

Thus,

a couple (or torque) acts on the electric dipole and as a result it rotates about its central axis which is given as,

couple = either force × ⊥ dist b/w two force.

as $\tau = qE(AK)$

$\tau = qE(AB \sin\theta)$

$= qE(2l \sin\theta)$

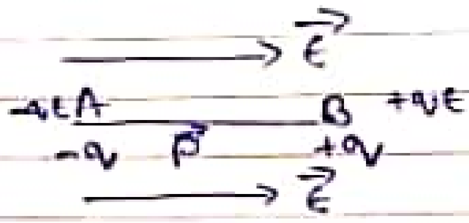
$\tau = p.E \sin\theta$

$$\boxed{\vec{\tau} = (\vec{p} \times \vec{E})}$$

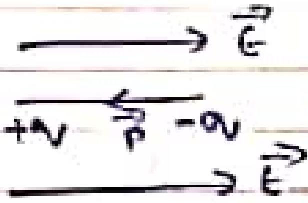
Special cases -

(i) If $\theta = 0^\circ \longrightarrow$ Stable equilibrium.
i.e. $\vec{p} \parallel \vec{E}$ i.e. $\sin 0^\circ = 0$

$$\boxed{\tau = 0}$$



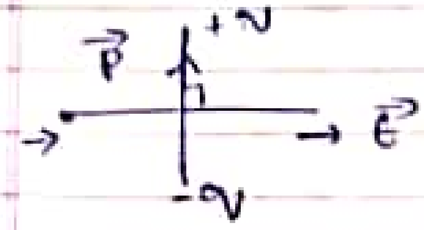
(ii) If $\theta = 180^\circ \longrightarrow$ unstable equilibrium
i.e. $\vec{p} \upuparrows \vec{E}$ $\sin 180^\circ = 0$



$$\boxed{\tau = 0}$$

(iii) If $\theta = 90^\circ$
i.e. $\vec{p} \perp \vec{E}$ $\sin 90^\circ = 1$

$\boxed{\tau = 1}$ – Max^m.



Instantly, The net torque (τ) acting on the dipole in a uniform $\vec{E}$ is

$$\tau = PE\sin\theta$$

for small angular displacement ($d\theta$), small work is

$$dw = \tau d\theta$$
$$= PE\sin\theta d\theta$$

$$W_{total} = \int_{\theta_1}^{\theta_2} dw = PE\int_{\theta_1}^{\theta_2} \sin\theta d\theta$$
$$= PE\left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$
$$= -PE(\cos\theta_2 - \cos\theta_1)$$

$$\boxed{U = W = PE(\cos\theta_1 - \cos\theta_2)}$$

If $\theta_1 = 90^\circ$ & $\theta_2 = \theta$

$$U = PE(\cos 90^\circ - \cos\theta)$$
$$= -PE\cos\theta$$
$$\boxed{U = -\vec{P}\cdot\vec{E}}$$

21

Q Two point charges $-0.2\ \mu C$ and $+0.2\ \mu C$ are respectively placed at two points A(2cm, 1cm, -1cm) and B(3cm, 2cm, 2cm) forming an electric dipole placed in an uniform electric field $\vec{E} = (-\hat{i} + 2\hat{j} - \hat{k})$ N/C. Calculate (i) Net torque experience by electric dipole. (ii) Total work done by the dipole on the condition of maximum ~~work~~ torque to the stable equilb^m?

22

Q-$\vec{\tau} = \vec{p} \times \vec{E}$

(i) $\tau = \vec{p} \times \vec{E}$

$\vec{p}$: $q(\vec{2l})$

$q(\vec{AB})$

$q\left[(2-1)\hat{i} + (2-1)\hat{j} + (-2+1)\hat{k}\right]$ cm

$0.2 \times 10^{-6} (\hat{i} + \hat{j} - \hat{k}) \times 10^{-2}$ Cm

$= 2 \times 10^{-3} (\hat{i} + \hat{j} - \hat{k})$ Cm

Reqd. torque $= \vec{p} \times \vec{E}$

$$\vec{\tau} = \begin{vmatrix} i & j & k \\ 2\times10^{-3} & 2\times10^{-3} & -2\times10^{-3} \\ -1 & 2 & -1 \end{vmatrix}$$

$$2\times10^{-3} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ -1 & 2 & -1 \end{vmatrix}$$

$2\times10^{-3}\left\{(-1+2)\hat{i} + (1+1)\hat{j} + (2+1)\hat{k}\right\}$

$2\times10^{-3}(\hat{i} + 2\hat{j} + 3\hat{k})$ Nm

(ii) Req^d. Energy stored or work done (W) from

$\theta_1 = 90^\circ$ (max^m torque) & $\theta_2 = 0^\circ$ (stable equlibm)

$$U = W = pE(\cos\theta_1 - \cos\theta_2)$$

$$= pE(\cos 90^\circ - \cos 0^\circ)$$

$$= -p \cdot E$$

$$|\vec{p}| = \sqrt{(1)^2 + (1)^2 + (1)^2} \; 2\times10^{-3}$$

$$= \sqrt{3} \cdot 2\times10^{-3}$$

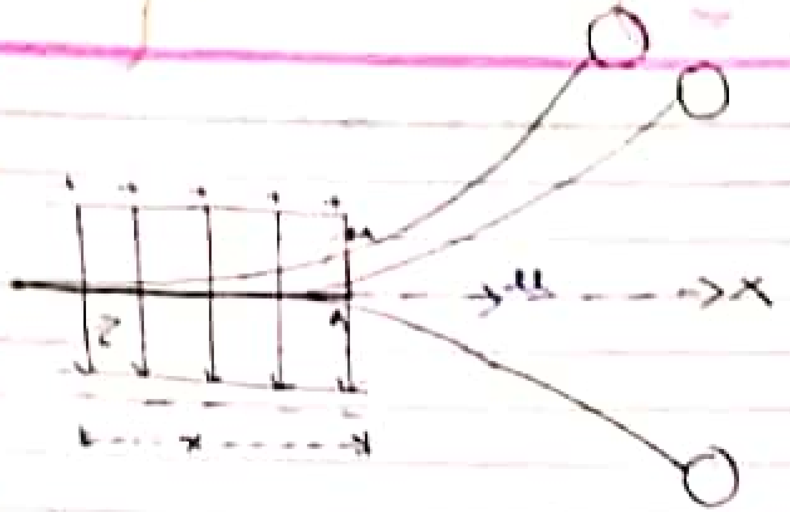
$$|\vec{E}| = \sqrt{(1)^2 + (2)^2 + (1)^2}$$

$$= \sqrt{6}$$

$$= -(2\times10^{-3})\sqrt{3}\cdot\sqrt{6}$$

$$= -2\sqrt{18}\times10^{-3}$$

$$= -6\sqrt{2}\times10^{-3}\ J$$



$\vec{u} \perp \vec{E}$

Horizontal motion

$u_x = u = \text{const.}$

$\therefore a_x = 0$

$x = u_x t + \frac{1}{2} a_x t^2 = ut$

$t = \frac{x}{u}$ — (10)

Verticle motion:

$u_y = 0 \neq \text{const.}$

$m a_y = qE$

$a_y = \frac{qE}{m}$

$y = u_y t + \frac{1}{2} a_y t^2$

$0 + \frac{1}{2}\frac{qE}{m}\left(\frac{x}{u}\right)^2$

$$\boxed{y = \frac{qE x^2}{2mu^2}}$$

25

$$\boxed{y = \left(\frac{qE^2}{2mu^2}\right)x^2} \quad (1)$$

It represent the eqn of the parabola ($y \propto x^2$)

Thus, the trajectory of the charged particle is parabolic in nature.

from (1)

(i)
$$\left(\frac{q}{m}\right) = \left(\frac{2u^2}{Ex^2}\right)y$$

$$\boxed{\left(\frac{q}{m}\right) \propto y}$$

(ii) $\because \left(\frac{q}{m}\right) \propto y$ (vertical displacement)

$\because y_1 > y_2 > y_3$

$\therefore \left(\frac{q}{m}\right)_1 > \left(\frac{q}{m}\right)_2 > \left(\frac{q}{m}\right)_3$ m

Q. A charge Q is divided into two parts q & (Q−q) are kept at a seperation. calculate the magnitude of each charge if the coulombs force between two charges is minimum/maximum.

(17)

$F = \frac{k\, q_1 q_2}{a^2}$

$= \frac{k}{a^2}\, q(Q-q)$

for the force (F) to be max^m & min.

$\frac{dF}{dq} = 0$

$\frac{k}{a^2}\{q(0-1) + (Q-q)1\} = 0$

$-q + Q - q = 0$

$2q = Q$

$q = Q/2$

$q_1 = q = Q/2$

$q_2 = Q - q = Q/2$