CHAPTER 1

Number Systems

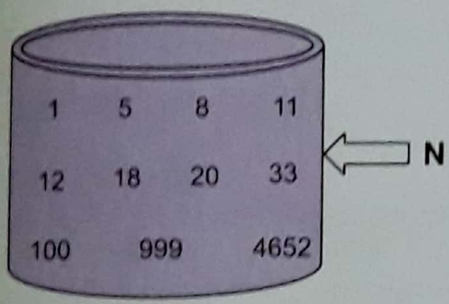
"Numbers are intellectual witnesses that belong only to mankind"

Introduction

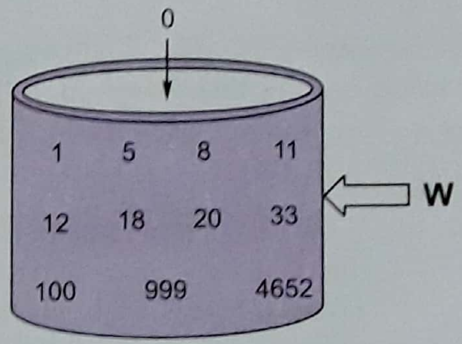
In our earlier classes, we have learnt about the various types of numbers and how to represent them on a number line. Let us first review some of those important concepts.

Natural Numbers

The counting numbers 1, 2, 3, ... etc ; are called natural numbers. This collection can be represented as $N = \{1, 2, 3, 4, \ldots\ldots\}$.



Bucket of natural numbers.



Adding 0 to the bucket of natural numbers.

Whole Numbers

All counting numbers together with 0 form the collection of whole numbers. Thus,

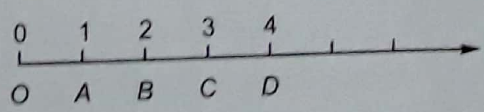
$$W = \{0, 1, 2, 3, 4, \ldots\ldots\}$$

REMARKS

1. Every natural number is a whole number.
2. 0 is a whole number but it is not a natural number.
3. 0 (zero) is the smallest whole number.

Representation of Whole Numbers on a Number Line

Draw a straight line and mark a point O on it. The point O corresponds to the number 0 (zero). Starting from O, mark points A, B, C, D, etc., at equal distances on the right of O. Then the points A, B, C, D, etc., respectively represent the numbers 1, 2, 3, 4, etc. This line is called a number line.



Clearly, the number line can be extended endlessly on the right. So we have an infinite number of whole numbers (or natural numbers).

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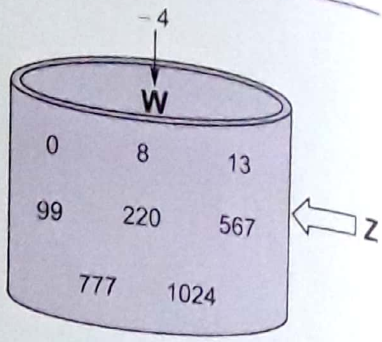
Integers



Adding negative counting numbers to the bucket of whole numbers

All counting numbers, the negative of counting numbers and 0 form the collection of integers. Thus,

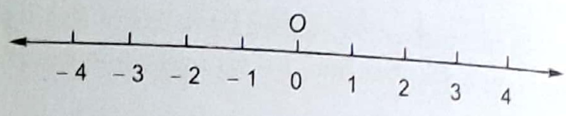
$$Z \text{ or } I = \{......, -3, -2, -1, 0, 1, 2, 3, ...\}$$

REMARKS

1. Every natural number is an integer.
2. Every whole number is an integer.
3. The collection of positive integers is $Z^{+} = \{1, 2, 3, 4, ...\}$
4. The collection of negative integers is
   $Z^{-} = \{......, -4, -3, -2, -1\}$
5. The symbol Z originated from the German word Zahlen, which means to count.

Representation of Integers on a Number Line

Draw a straight line and mark a point O almost in the middle of it. Let this point represent the integer 0 (zero). Now, set off equal distances to the right as well as to the left of O.



On the right side of O, the points of division respectively represent the integers 1, 2, 3, 4, etc.

Similarly, on the left side of O, the points of division respectively represent the integers $-1, -2, -3, -4$, etc.

As the number line can be extended endlessly on both sides of O, so we can represent each and every integer on this line. For example, if we start from O and move to right a distance of 555 units, we get a point which represents the integer 555.

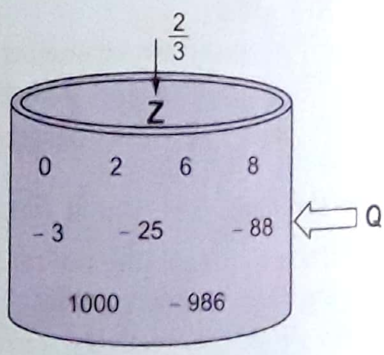
REMARKS

1. If we represent two integers on the number line, then the integer on the right is greater than the integer on the left. If an integer a lies to the left of the integer b, we say that $a < b$. Likewise, if b lies to the right a, we say that $b > a$. For example, $4 > 2, -1 > -3, -4 < -2$, etc.
2. Every positive integer is greater than every negative integer.
3. Zero is less than every positive integer and greater than every negative integer.
4. For integers a and b, if $a < b$, then $-a > -b$. For example, $3 < 7 \Rightarrow -3 > -7$.

We shall see that there are some more types of numbers still left which can be represented on a number line.

Rational Numbers

Any number r that can be expressed in the form $\frac{p}{q}$, where p, q are integers and $q \neq 0$, is called a rational number. The collection of rational numbers is denoted by Q. Rational comes from the word ratio, while Q comes from the word quotient.



Adding fractions to the bucket of integers.

REMARKS

1. Every natural number is a rational number, as we can write $1 = \frac{1}{1}, 2 = \frac{2}{1}, 3 = \frac{3}{1}$, etc.
2. 0 is a rational number, as we can write $0 = \frac{0}{1}$.
3. Every integer is a rational number, as we can write $4 = \frac{4}{1}, -4 = \frac{-4}{1}, -8 = \frac{-8}{1}$, etc.

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Equivalent Rational Numbers or Fractions

Rational numbers do not have a unique representation in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. *For example,*

$$\frac{1}{2}=\frac{2}{4}=\frac{5}{10}=\frac{25}{50}=\frac{42}{84}, \text{ and so on.}$$

These are equivalent rational numbers or fractions.

Rational Number in Lowest Terms

A rational number $\frac{p}{q}$ is said to be in lowest terms (or simplest form) if the integers p and q have no common factors other than 1 (that is, p and q are co-prime) and $q \neq 0$.

The fraction $\frac{8}{12}$ in lowest terms is $\frac{2}{3}$.

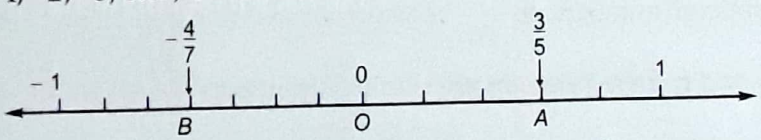
The fraction $\frac{19}{95}$ in lowest terms is $\frac{1}{5}$.

Among the infinitely many fractions equivalent to $\frac{p}{q}$, we always choose the simplest form of $\frac{p}{q}$ to represent on the number line all such equivalent fractions.

Representation of a Rational Number on a Number Line

Example : *Represent (i) $\frac{3}{5}$ and (ii) $-\frac{4}{7}$ on the number line.*

Solution. Draw a straight line. Mark a point O on it, which represents number 0. Mark points by setting off equal distances to the right as well as to the left of O. Label these points as 1, 2, 3, etc. ; on the right and −1, −2, −3, etc. ; on the left of O.



(*i*) In $\frac{3}{5}$, numerator is less than denominator, so $\frac{3}{5}<1$. Also, $\frac{3}{5}$ is a positive rational number. Divide the segment between 0 and 1 into 5 equal parts. Starting from 0, count 3 parts to the right to get the point A. It represents the rational number $\frac{3}{5}$.

(*ii*) Now, $-\frac{4}{7}$ is a negative rational number and its numerical value is less than 1. Divide the segment between 0 and 1 into 7 equal parts. Starting from 0, count 4 parts to the left to get a point B. It represents the rational number $-\frac{4}{7}$.

Example : *Represent (i) $\frac{11}{4}$ and (ii) $-\frac{9}{5}$ on the number line.*

Solution. (*i*) $\frac{11}{4}=2\frac{3}{4}$

Clearly, $\frac{11}{4}$ lies between 2 and 3. As shown in the figure on next page, divide the segment between 2 and 3 into 4 equal parts. Starting from 2, count 3 parts to the right to get point P. It represents the rational number $\frac{11}{4}$.

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