Example:- Represent $\frac{11}{4}$ and $-\frac{9}{5}$ on the number line.

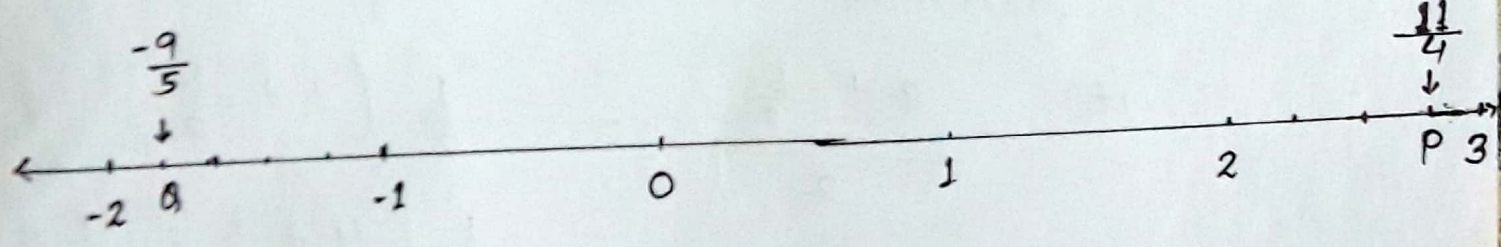
Solution:

$$\frac{11}{4} = 2\frac{3}{4}$$

clearly $\frac{11}{4}$ lies between 2 and 3.

As the previous example, divide the segment between 2 and 3 into 4 equal parts. Starting from 2, count 3 parts to the right to get point P. It represent the rational number $\frac{11}{4}$.

Also, $-\frac{9}{5} = -1\frac{4}{5}$. clearly $-\frac{9}{5}$ lies between -1 and -2. Divide the segment between -1 and -2 into 5 equal parts. Starting from -1, count 4 parts to the left to get point Q. It represent the rational number $-\frac{9}{5}$.

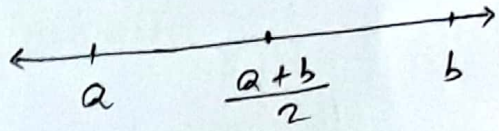


* To Find a rational number between two given rational number.

Between two distinct rational numbers a and b, there exists another rational number,

Let $a < b$

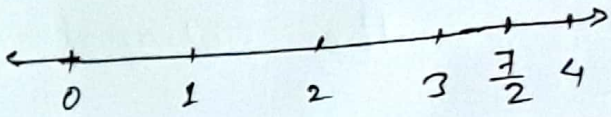
Then a rational number between a and b is $\frac{a+b}{2}$.



example:- Find a rational number between 3 and 4.

Soln:- A rational number between 3 and 4 is

$$\frac{3+4}{2} = \frac{7}{2} = 3.5$$



example:- Find three rational number between 2 and 3.

Soln:- A rational number between 2 and 3 is $\frac{2+3}{2} = \frac{5}{2}$.

∴ Now, we take 2 and $\frac{5}{2}$.

By the same process, we can find a rational number between 2 and $\frac{5}{2}$

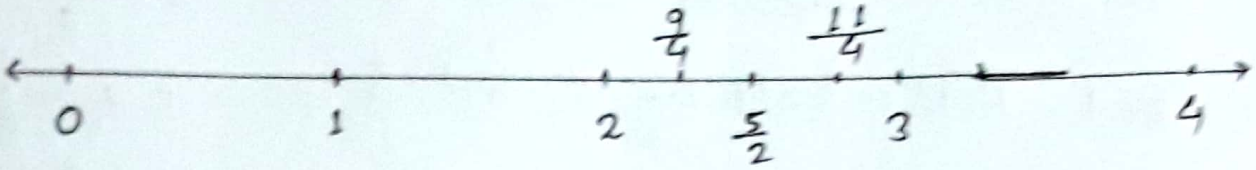
ie $$\frac{2+\frac{5}{2}}{2} = \frac{1}{2}\left(2+\frac{5}{2}\right)$$

$$= \frac{1}{2}\left(\frac{4+5}{2}\right) = \frac{1}{2}\times\frac{9}{2} = \frac{9}{4}$$

Similarly, A rational number between 3 and $\frac{5}{2}$ is $\frac{3+\frac{5}{2}}{2} = \frac{1}{2}\left(3+\frac{5}{2}\right)$

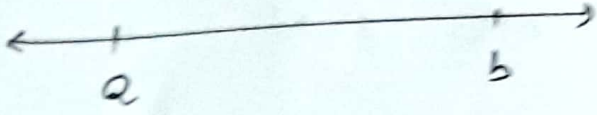
$= \frac{1}{2}\left(\frac{6+5}{2}\right)$

$= \frac{1}{2} \times \frac{11}{2} = \frac{11}{4}$



* To Find a Rational number between two given Rational number a and b.

Let $a < b$



Then $d = \frac{b-a}{n+1}$

∴ Ist Rational Number between a and b is $a+d$.

2nd R.N is $a+2d$.

3rd R.N is $a+3d$

⋮ ⋮ ⋮ ⋮

nth Rational number is $a+nd$.

Thus, n rational number between a and b will be

$a+d, a+2d, a+3d, \ldots, a+nd$.

Ex:- Find Six Rational number between 5 and 6.

Soln:- Let $a = 5$ and $b = 6$

Then $d = \frac{b-a}{n+1} = \frac{6-5}{6+1} = \frac{1}{7}$

Ist R.N between 5 and 6 is $5 + \frac{1}{7} = \frac{36}{7}$

2nd R.N between 5 and 6 is $5 + 2\times\frac{1}{7}$

$= 5 + \frac{2}{7} = \frac{37}{7}$

3rd R.N " 5 and 6 is $5 + 3\times\frac{1}{7}$

$= 5 + \frac{3}{7} = \frac{38}{7}$

4th R.N " 5 and 6 is $5 + 4\times\frac{1}{7}$

$= 5 + \frac{4}{7} = \frac{39}{7}$

5th R.N " 5 and 6 is $5 + 5\times\frac{1}{7}$

$= 5 + \frac{5}{7} = \frac{40}{7}$

6th R.N " 5 and 6 is $5 + 6\times\frac{1}{7}$

$= 5 + \frac{6}{7} = \frac{41}{7}$

∴ Six Rational no. are $\frac{36}{7}, \frac{37}{7}, \frac{38}{7}, \frac{39}{7}, \frac{40}{7}, \frac{41}{7}$.

Ex:- Find 5 rational number between $\frac{2}{5}$ and $\frac{3}{4}$.

Soln:- Since $\frac{2}{5} < \frac{3}{4}$

Let $a = \frac{2}{5}$ and $b = \frac{3}{4}$

Then $d = \frac{b-a}{n+1} = \frac{\frac{3}{4} - \frac{2}{5}}{5+1} = \frac{\frac{15-8}{20}}{6}$

$= \frac{1}{6}\left(\frac{7}{20}\right) = \frac{1}{6} \times \frac{7}{20} = \frac{7}{120}$

Thus, five Rational number between $\frac{2}{5}$ and $\frac{3}{4}$ are

$a+d, \quad a+2d, \quad a+3d, \quad a+4d, \quad a+5d$

or, $\frac{2}{5}+\frac{7}{120}, \frac{2}{5}+2\times\frac{7}{120}, \frac{2}{5}+3\times\frac{7}{120}, \frac{2}{5}+4\times\frac{7}{120}$

and $\frac{2}{5}+5\times\frac{7}{120}$

or, $\frac{2}{5}+\frac{7}{120}, \frac{2}{5}+\frac{14}{120}, \frac{2}{5}+\frac{21}{120}, \frac{2}{5}+\frac{28}{120},$

and $\frac{2}{5}+\frac{35}{120}$

or, $\frac{48+7}{120}, \frac{48+14}{120}, \frac{48+21}{120}, \frac{48+28}{120}$

and $\frac{48+35}{120}$

or, $\frac{55}{120}, \frac{62}{120}, \frac{69}{120}, \frac{76}{120}, \frac{83}{120}$

Alternative Method :- First we make the given rational into equivalent Rational Number.

$\frac{2}{5}$ and $\frac{3}{4}$. LCM of 5 and 4 is 20

$\therefore \frac{2}{5} = \frac{2\times4}{5\times4} = \frac{8}{20}$

$\frac{3}{4} = \frac{3\times5}{4\times5} = \frac{15}{20}$

Thus, five rational number between $\frac{2}{5}$ and $\frac{3}{4}$ are

$$\frac{9}{20}, \frac{10}{20}, \frac{11}{20}, \frac{12}{20}, \frac{13}{20}$$

Check Your Progress:-

(1.) State true or false. Also give reasons for your answers.

(a) Every Rational number is a whole number.

(b) Every integer is a rational number.

(c) Every Rational number is an integer.

(2) Find Six Rational number between 3 and 4.

(3) Represent the following on the number line

(i) $\frac{5}{7}$ (ii) $\frac{13}{4}$ (iii) $\frac{-8}{3}$ (iv) $\frac{-4}{9}$

(4) Find a rational number between $\frac{1}{4}$ and $\frac{1}{3}$.

(5) Find three Rational number between $\frac{1}{6}$ and $\frac{1}{5}$

(6) Find ten Rational number between $-\frac{1}{9}$ and $\frac{4}{9}$

(7) Find Eight Rational number between $\frac{2}{7}$ and $\frac{3}{5}$.