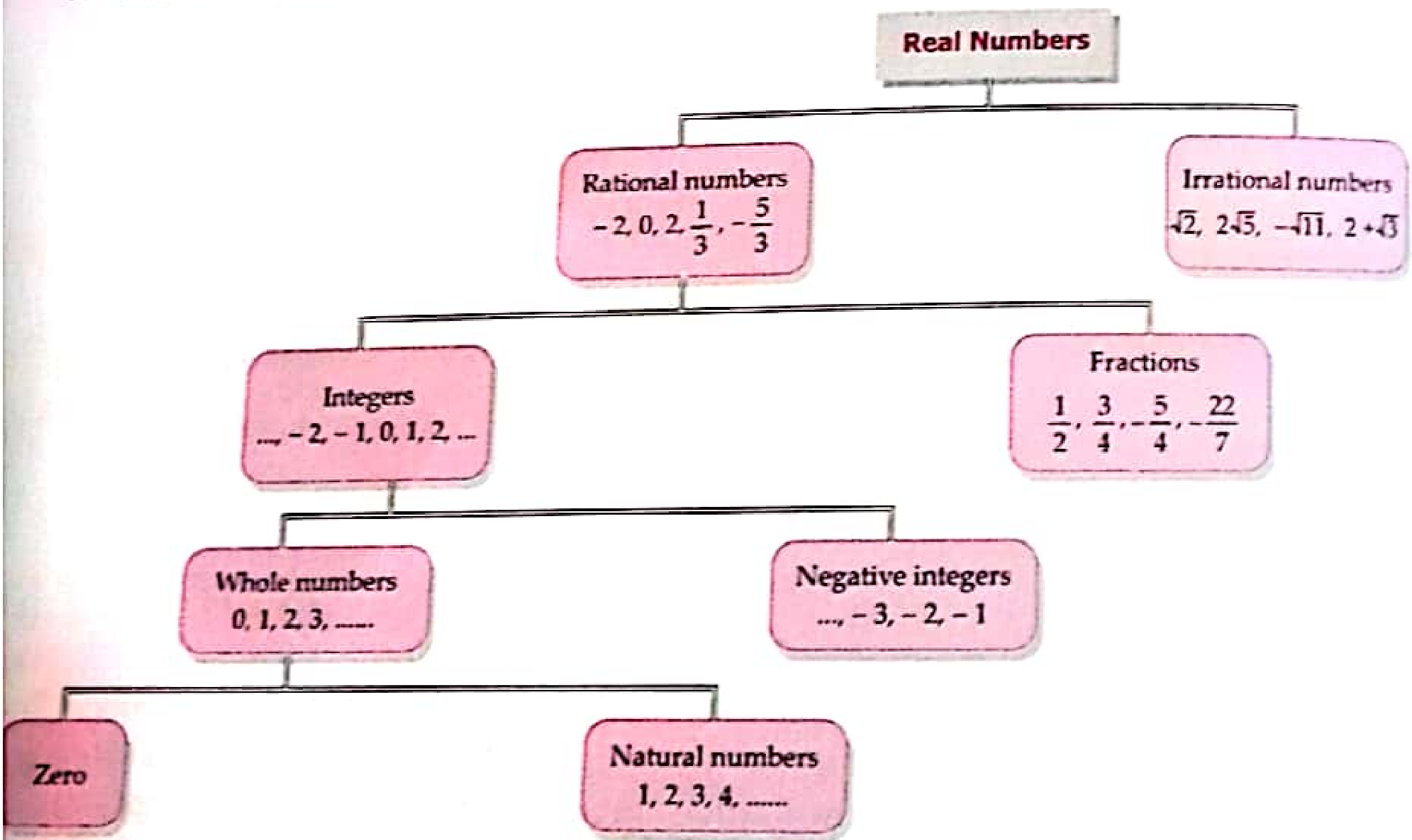
Real numbers

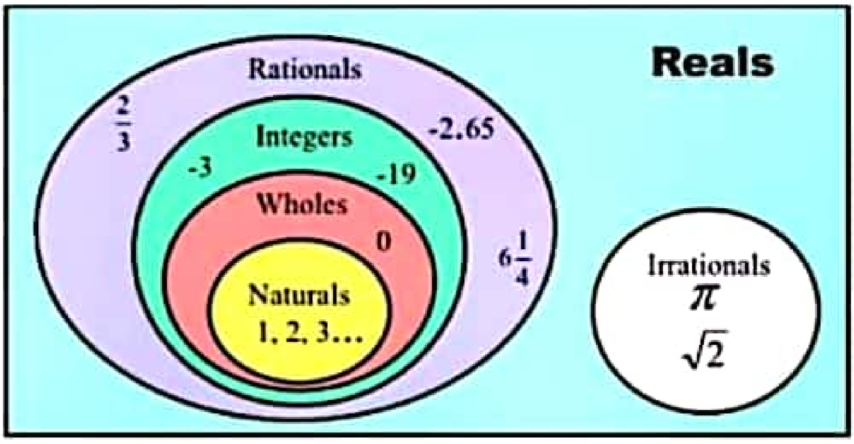
All rational numbers and all irrational numbers together form the collection of real numbers.



REAL NUMBERS

INTRODUCTION

In class IX , you have already study about real numbers and encountered irrational numbers.



Here we discuss about very important properties of positive integers Euclid's division algorithm and fundamental theorem of arithmetic .Euclid's division algorith mainly use to compute the H.C.F of two positive integers.On the other hand the fundamental thereom of arithmetic is some thing related to multiplication of positive integers .First we begin with

Euclid's Division Lemma

Consider the following folk puzzel

A trader was moving along a road selling eggs. An idler who didn't have much work to do, started to get the trader into a wordy duel. This grew into a fight, he pulled the basket with eggs and dashed it on the floor. The eggs broke. The trader requested the Panchayat to ask the idler to pay for the broken eggs. The Panchayat asked the trader how many eggs were broken. He gave the following response:

If counted in pairs, one will remain;

If counted in threes, two will remain;

If counted in fours, three will remain;

If counted in fives, four will remain;

If counted in sixes, five will remain;

If counted in sevens, nothing will remain;

My basket cannot accomodate more than 150 eggs.

So, how many eggs were there?

Let us try to solve the puzzel.

Let the number of eggs be a. Then working backwards, we see that a is less than or equal to 150.

According to the trader

a=7m+0, a=6n+5 ,a=5p+4,a=4q+3, a=3u+2,a=2v+1,Where m,n,p,q,u and v are natural numbers

You must look for the multiple of of 7 which satisfy the all conditions.

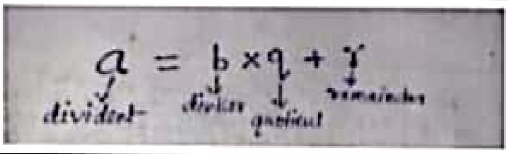
The answer is 119

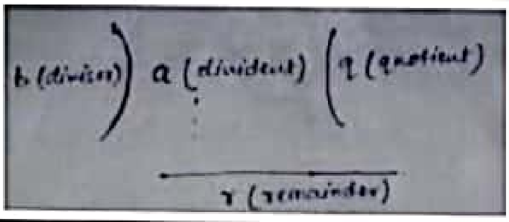
119 (119=7*17+0,119=6*19+5,119=5*23+4,119=4*29+3,119=3*39+2,119=2*59+1)

Thus we see that in each cases we have a (dividend) and a positive integer b (divisor) ,there exist unique integers q (quotient) , b divide a and gives remainder r less than b.This is nothing but Eclud's division lemma.

Euclid's Division Lemma

Given integers a and b , there exist unique integers q and r satisfying a=bq+r, $0 \leq r \leq b$

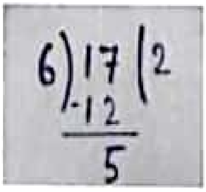




Example: Let pairs of integers 17 and 6

we can write

17=6×2 + 5



Note(a) A lemma is proven statement used for proving another statement

(b) An algorithm is series of well defined steps which gives a procedure for solving a type of problem.

(c) Euclid's division lemma is nothing but a restatement of the long division process.

Euclid's Division Algorithm

It is a technique to compute the Highest Common Factor(HCF).

To calculate the HCF of two positive integers a and b with a > b, the following steps are followed:

Step 1: Apply Euclid's division lemma to find q and r where a = bq + r, $0 \leq r < b$.

Step 2: If the remainder i.e. r = 0, then the HCF will be 'b' but if $r \neq 0$ then we have to apply Euclid's division lemma to b and r.

Step 3: Continue with this process until we get the remainder as zero. Now the divisor at this stage will be HCF(a, b). Also, HCF (a, b) = HCF (b, r), where HCF (a, b) means HCF of a and b.

EXAMPLE 1: Use Euclid's algorithmto find the HCF of 420 and 66.

Solution: Here 420>66. Now by Euclid's division algorithm

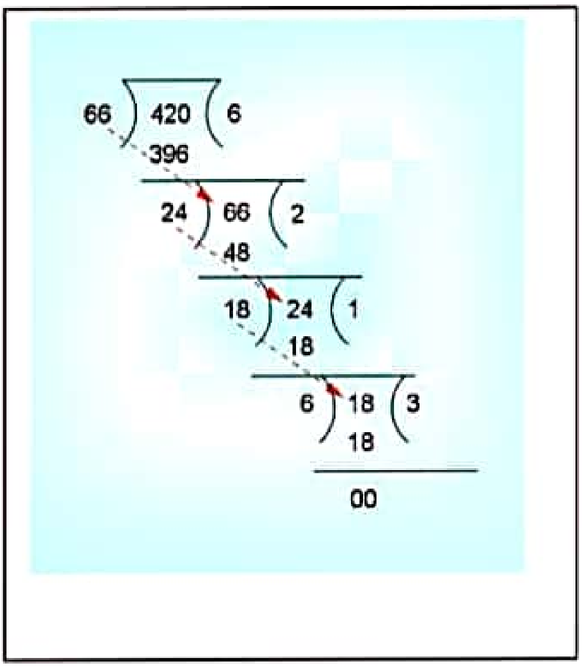
Step 1: $420 = 66 \times 6 + 24$

Step 2: $66 = 24 \times 2 + 18$

Step 3: $24 = 18 \times 1 + 6$

Step 4: $18 = \boxed{6} \times 3 + 0$

Therefore HCF(420,66)=6



EXAMPLE 2 : Show that any positive odd integer is of the form 4q + 1 or 4q + 3, where q is some integer. Solution : Let a is a positive odd integer and b=4.

By Euclid's division algorithm

$a = 4q + r$, for some integer $q \geq 0$ and $r=0,1,2,3$.

For r=0 ,a=4q ;For r=1, a=4q+1;For r=2, a=4q+2;For r=3,a=4q+3

Since a is odd, a cannot be 4q or 4q + 2 (since they are both divisible by 2).

Therefore, any odd integer wii be of the form 4q + 1 or 4q + 3.

EXAMPLE 3: Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

Solution: Let a be any positive integer and b=3

By Euclid's division algorithm

$a = 3q + r$, for some integer $q \geq 0$ and $r=0,1,2$,

Case 1: When r=0 $a = 3q$ $a^2 = (3q)^2 = 9q^2$ $= 3.3q^2$ $= 3m$ (assuming $m=3q^2$)	Case 2: When r=1 $a = 3q+1$ $a^2 = (3q+1)^2 = 9q^2 + 6q + 1$ $= 3.(3q^2+2q)+1$ $= 3m+1$ (assuming $m=3q^2+2q$)	Case 1 When r=2 $a = 3q+2$ $a^2 = (3q+2)^2 = 9q^2 + 12q + 4$ $= 9q^2+12q+3+1$ $=3(3q^2+4q+1)+1$ $= 3m+1$ (assuming $m=3q^2+4q+1$)

Therefore, the square of any positive integer is always of the form of 3m or 3m+1 .Hence Proved

∴ HCF (867, 255) = 51.

Example 2 Use Euclid's division algorithm to find the HCF of 10224 and 9648. [CBSE 2011]

Solution. Here $10224 > 9648$.

We apply Euclid's division lemma to 10224 and 9648 repeatedly till the remainder becomes zero.

We get

$$10224 = 9648 \times 1 + 576$$

$$9648 = 576 \times 16 + 432$$

$$576 = 432 \times 1 + 144$$

$$432 = 144 \times 3 + 0$$

```
9648)10224(1
     9648
     ----
      576)9648(16
          9216
          ----
           432)576(1
               432
               ---
               144)432(3
                   432
                   ---
                    0
```

As the last divisor is 144, so

HCF (10224, 9648) = 144.

REMARK HCF of three numbers is the HCF of the HCF of any two of them and the third number.

Example 3 Use Euclid's division algorithm to find the HCF of 441, 567, 693.

[Exemplar Problem]

Solution. Given three numbers 441, 567, 693.

We first find HCF of 441 and 567.

By division algorithm, we have

$$567 = 441 \times 1 + 126$$

$$441 = 126 \times 3 + 63$$

$$126 = 63 \times 2 + 0$$

$\Rightarrow$ HCF (441, 567) = 63

```
441)567(1
    441
    ---
    126)441(3
        378
        ---
         63)126(2
            126
            ---
             0
```

Now we find HCF of 693 and 63.

We have

$$693 = 63 \times 11 + 0$$

$\Rightarrow$ HCF (693, 63) = 63

Hence,

HCF (441, 567, 693) = 63.

```
63)693(11
   63
   --
    63
    63
    --
     0
```

CHECK YOUR PROGRESS :.

1. Use Euclid's division algorithm to find the HCF of (i) 867 and 255 (ii) 960 and 432.

2. Show that any positive odd integer is of the form $6q+1$, $6q+3$, or $6q+5$ where q is some integer

3. Show that the square of any positive ~~integer~~ odd integer is of the form $8m+1$ for some integer m.

4. Show that the cube of any positive integer is of the form $9m$, $9m+1$ or $9m+8$ for some integer m.