Secondary

MATHEMATICS

Class-VII

Publication Division

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Contents

CHAPTER 1

RATIONAL NUMBERS

INTRODUCTION

Do you remember number systems?

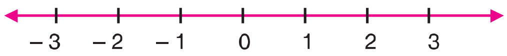
(i) Numbers 1, 2, 3, 4, . . . which we use for counting, form the **system of natural numbers.**

Natural numbers are 1, 2, 3, 4 ....

(ii) Natural numbers along with zero, form the **system of whole numbers.**

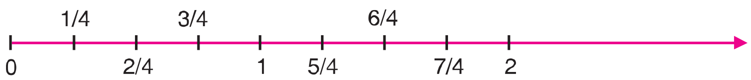
Whole numbers are 0, 1, 2, 3, 4, 5..........

(iii) Collection of natural numbers, their opposites along with zero is called the **system of integers.**



Integers are ...., – 3, – 2, – 1, 0, 1, 2, 3 ....

(iv) A part of a whole is a fraction. **Fraction** is the ratio of two natural numbers, e.g. $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{5}{4}, \frac{6}{4}, \ldots$



Properties of fractions

(a) If $\frac{p}{q}$ is a fraction, then for any natural number m,

$$\frac{p}{q} = \frac{p \times m}{q \times m}$$

(b) If $\frac{p}{q}$ is a fraction and a natural number m is a common divisor of p and q, then

$$\frac{p}{q} = \frac{p \div m}{q \div m}$$

(c) Two fractions $\frac{p}{q}$ and $\frac{r}{s}$ are said to be equivalent if

$$p \times s = q \times r$$

(d) A fraction $\frac{p}{q}$ is said to be in its simplest or lowest form if

p and q have no common factor other than 1.

(e) Fractions can be compared as:

(i) $\frac{p}{q} < \frac{r}{s}$ if $p \times s < q \times r$

(ii) $\frac{p}{q} = \frac{r}{s}$ if $p \times s = q \times r$

(iii) $\frac{p}{q} > \frac{r}{s}$ if $p \times s > q \times r$

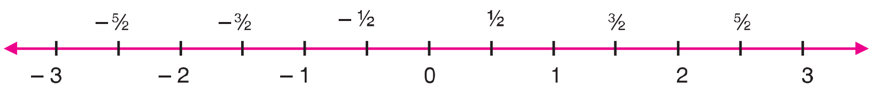
Let us do some problems to revise our memory.

Simplify the following:

1. $(-212) + 384 - (-137)$
2. $(-9) \times [7 + (-11)]$
3. $(-12) \times (-10) \times 6 \times (-1)$
4. $(-108) \div (-12)$
5. $(-1331) \div 11$
6. $-72\ (-15 - 37 - 18)$

RATIONAL NUMBERS

In Class–VI, we have dealt with negative integers. In the same way, we shall be introducing negative fractions, e.g. corresponding to 1/2 we have negative fraction – 1/2.



Fractions with corresponding negative fractions and zero constitute the **system of rational numbers.**

The word ‘rational’ comes from the word ‘ratio’.

Any number which can be expressed in the form of p/q, where p and q are integers and $q \neq 0$ is known as a **Rational Number.**

See the following rational numbers.

$$\frac{1}{5} \quad \frac{5}{-2} \quad \frac{2}{3} \quad \frac{-1}{5} \quad \frac{-2}{3} \quad \frac{-2}{-3} \quad \frac{1}{-4}$$

Positive Rational Numbers

$$\frac{1}{5}, \frac{2}{3}, \frac{-2}{-3}$$

The rational numbers are said to be **positive** if signs of numerator and denominator are the same.

Negative Rational Numbers

$$\frac{5}{-2}, \frac{-1}{5}, \frac{-2}{3}, \frac{1}{-4}$$

The rational numbers are said to be **negative** if signs of numerator and denominator are not the same.

Remember

- Every fraction is a rational number, but every rational number need not be a fraction, e.g. $\frac{-4}{7}, \frac{0}{3}$ are not fractions as fractions are part of a whole which are always positive.
- All the integers are rational numbers. Integers – 50, 15, 0 can be written as, $\frac{-50}{1}, \frac{15}{1}, \frac{0}{1}$ respectively.

Worksheet 1

1. **Which of the following are rational numbers?**

   (i) -3 (ii) $-\frac{2}{3}$ (iii) $\frac{4}{0}$ (iv) $\frac{0}{-5}$

2. **Write down the rational numbers in the form $\frac{p}{q}$ whose numerators and denominators are given below:**

   (i) $(-5) \times 4$ and $-5 + 4$ (ii) $64 \div 4$ and $32 - 18$

3. Which of the following are positive rational numbers?

(i) $\frac{-2}{9}$ (ii) $\frac{3}{-5}$ (iii) $\frac{4}{9}$ (iv) $\frac{-3}{-19}$ (v) $\frac{0}{-3}$

4. Answer the following:

(i) Which integer is neither positive nor negative?

(ii) A rational number can always be written as $\frac{p}{q}$. Is it necessary that any number written as $\frac{q}{p}$ is a rational number?

5. State whether the following statements are true. If not, justify your answer with an example.

(i) Every whole number is a natural number.

(ii) Every natural number is an integer.

(iii) Every integer is a whole number.

(iv) Every integer is a rational number.

(v) Every rational number is a fraction.

(vi) Every fraction is a rational number.

PROPERTIES OF RATIONAL NUMBERS

Property I. Two rational numbers $\frac{p}{q}$ and $\frac{r}{s}$ are said to be **equivalent** if

$$p \times s = r \times q.$$

To explain the property, let us take few examples.

Example 1: Show that $\frac{4}{-7}$ and $\frac{8}{-14}$ are equivalent rational numbers.

Solution: $4 \times (-14) = -56 = 8 \times (-7)$.

Hence, $\frac{4}{-7}$ and $\frac{8}{-14}$ are equivalent rational numbers.

Example 2: Show that $\frac{5}{8}$ and $\frac{-15}{24}$ are not equivalent rational numbers.

Solution: $5 \times 24 = 120$ and $8 \times (-15) = -120$.

Hence, $5 \times 24 \neq 8 \times (-15)$.

Therefore, the given rational numbers are not equivalent.

Property II. If $\frac{p}{q}$ is a rational number and m be any integer different from zero, then

$$\frac{p}{q} = \frac{p \times m}{q \times m}.$$

Example 3: Write three rational numbers which are equivalent to $\frac{3}{5}$.

Solution: To find equivalent rational numbers, multiply numerator and denominator by any same non-zero integer.

$$\frac{3 \times 2}{5 \times 2} = \frac{6}{10}$$ (Multiply numerator and denominator by 2)

$$\frac{3 \times (-3)}{5 \times (-3)} = \frac{-9}{-15}$$ (Multiply numerator and denominator by – 3)

$$\frac{3 \times 5}{5 \times 5} = \frac{15}{25}$$ (Multiply numerator and denominator by 5)

Hence, $\frac{6}{10}, \frac{-9}{-15}$ and $\frac{15}{25}$ are three rational numbers equivalent to $\frac{3}{5}$.

Example 4: Express $\frac{-4}{7}$ as a rational number with (i) numerator 12 (ii) denominator 28.

Solution: (i) To get numerator 12, we must multiply – 4 by – 3.

Hence, $$\frac{(-4) \times (-3)}{7 \times (-3)} = \frac{12}{-21}$$

Therefore, the required rational number is $\frac{12}{-21}$.

(ii) To get denominator 28, we must multiply the given denominator 7 by 4.

i.e. $$\frac{(-4) \times 4}{7 \times 4} = \frac{-16}{28}$$

Hence, the required rational number is $\frac{-16}{28}$.

Property III. If $\frac{p}{q}$ is a rational number and m is a common divisor of p and q then

$$\frac{p}{q} = \frac{p \div m}{q \div m}$$

Example 5: Express $\frac{-21}{49}$ as a rational number with denominator 7.

Solution: To get denominator 7, we must divide 49 by 7.

Therefore, $\frac{-21 \div 7}{49 \div 7} = \frac{-3}{7}$.

Hence, $\frac{-3}{7}$ is the required rational number.

Worksheet 2

1. In each of the following cases, show that the rational numbers are equivalent.

(i) $\frac{4}{9}$ and $\frac{44}{99}$ (ii) $\frac{7}{-3}$ and $\frac{35}{-15}$ (iii) $\frac{-3}{5}$ and $\frac{-12}{20}$

2. In each of the following cases, show that rational numbers are not equivalent.

(i) $\frac{4}{9}$ and $\frac{16}{27}$ (ii) $\frac{-100}{3}$ and $\frac{300}{9}$ (iii) $\frac{3}{-17}$ and $\frac{8}{-51}$

3. Write three rational numbers, equivalent to each of the following:

(i) $\frac{4}{7}$ (ii) $\frac{36}{108}$ (iii) $\frac{-5}{-7}$ (iv) $\frac{-72}{180}$

4. Express $\frac{3}{5}$ as rational number with numerator,

(i) – 21 (ii) 150

5. Express $\frac{4}{-7}$ as a rational number with denominator,

(i) 84 (ii) – 28

6. Express $\frac{90}{216}$ as a rational number with numerator 5.

7. Express $\frac{-64}{256}$ as a rational number with denominator 8.

8. Find equivalent forms of the rational numbers having a common denominator in each of the following collections of rational numbers.

(i) $\frac{2}{5}, \frac{6}{13}$ (ii) $\frac{1}{7}, \frac{2}{8}, \frac{3}{14}$ (iii) $\frac{5}{12}, \frac{7}{4}, \frac{9}{60}, \frac{11}{3}$

STANDARD FORM OF A RATIONAL NUMBER

Let us try to express a rational number in the simplest form with positive denominator.

Example 6: Express $\frac{16}{-24}$ in the simplest form with its denominator as positive.

Solution: **Step 1.** Convert denominator into positive by multiplying numerator and denominator by –1.

$$\frac{(16)\times(-1)}{(-24)\times(-1)} = \frac{-16}{24}$$

Step 2. Find HCF of 16 and 24, which is 8 in this case, and divide numerator and denominator by it.

$$\frac{-16 \div 8}{24 \div 8} = \frac{-2}{3}$$

The example given above explains that every rational number $\frac{p}{q}$ can be put in the simplest form with positive denominator. This form of the rational number is called its **standard form.** For this, we take the following steps.

Step 1. Make the denominator positive.

Step 2. Find the HCF m of p and q. If m = 1, then $\frac{p}{q}$ is the required form.

Step 3. If m ≠ 1, then divide both the numerator and the denominator by m. The rational number $\frac{p \div m}{q \div m}$ so obtained is the required standard form.

Note:

The numbers $\frac{-p}{q}$ and $\frac{p}{-q}$ represent the same rational number.

A rational number $\frac{p}{q}$ is said to be in the standard form if q is positive and the integers 'p' and 'q' have their highest common factor as 1.

Example 7: Express $\frac{-22}{-55}$ in the standard form.

Solution: **Step 1.** $\frac{-22\times(-1)}{-55\times(-1)} = \frac{22}{55}$

Step 2. HCF of 22 and 55 is 11.

$\frac{22\div 11}{55\div 11} = \frac{2}{5}$ which is the standard form.

Example 8: Find x such that the rational numbers in each of the following pairs are equivalent.

(i) $\frac{x}{12}, \frac{5}{6}$ (ii) $\frac{15}{x}, \frac{-3}{8}$

Solution: (i) $\frac{x}{12}, \frac{5}{6}$ will be equivalent if

$$6 \times x = 5 \times 12$$

$$x = \frac{5\times \cancel{12}^{2}}{\cancel{6}} = 5 \times 2 = 10$$

Hence, $x = 10$.

(ii) $\frac{15}{x}, \frac{-3}{8}$ will be equivalent if

$$15 \times 8 = (-3) \times x$$

$$x = \frac{15\times 8}{-3} = -5 \times 8 = -40$$

Hence, $x = -40$.

Example 9: Fill in the blanks: $\frac{-3}{5} = \frac{6}{\_\_\_} = \frac{\_\_\_}{-15}$

Solution: In the first two given rational numbers, we have to find the number which when multiplied by – 3 gives the product 6. Here, the number shall be 6 ÷ (– 3) = – 2. Now, we multiply both numerator and denominator of the given rational number by – 2.

We get $\frac{-3}{5} = \frac{(-3)\times(-2)}{5\times(-2)} = \frac{6}{-10}$